****

Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**MATHEMATICS SPECIALIST 3 & 4**

**Test 6 2018**

**Calculator-assumed**

Reading Time: 50 minutes

Time Allowed: 3 minutes Total Marks: 50

**Question 1 (4 marks)**

A particle moves in a straight line and, at time , the displacement from a fixed origin is , while denotes the velocity.

If the particle moves in the region , and , find the acceleration when .

|  |
| --- |
| **Solution** |
| So as  When , so  m/s2 |
| **Specific behaviours** |
| ✓ Sets up equation to find where v = 4  ✓ Determines that x = 0  ✓ Determines expression for acceleration in terms of x  ✓ Determines acceleration when v = 4.  dt |

**Question 2 (5 marks)**

An object, initially at rest, is dropped from the top of tall building so that after seconds it has velocity meters per second.

The air resistance encountered by the object is proportional to its velocity, so that the velocity satisfies the equation , where is a constant.

(a) Express the velocity of the object in terms of and . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ Separates variables  ✓ Integrates both sides  ✓ Determines expression for c  ✓ Determines expression for v |

(b) Sensors on the object indicate that its velocity will never exceed 55 metres per second. Determine the value of the constant . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ Determines value of k |

**Question 3 (6 marks)**

Kelly and Patsy meet up once a week for coffee. The time () in minutes that Kelly arrives later than Patsy is uniformly distributed with . The population mean is and the population variance is .

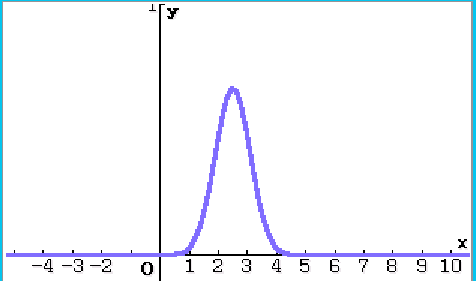
Patsy decides to keep a record of how late (or early) Kelly is over the course of a year (52 meet-ups). She determines the sample mean for this data.

(a) Determine correct to 2 decimal places. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ States that the sample mean is normally distributed  ✓ States the parameters of the sample mean  ✓ Calculates the probability |

(b) If a large number of samples, each with 52 meet-ups, is taken, sketch the likely distribution of the sample mean below.

In the diagram indicate or refer to the calculation from part (a) (3 marks)



|  |
| --- |
| **Solution** |
| As above. There is approximately 60% of the total area under the curve between |
| **Specific behaviours** |
| ✓ Indicates a normal distribution centred at  ✓ Indicates a standard deviation of just over  ✓ Refers to the probability from part (a) |

**Question 4 (6 marks)**

(a) The position of a particle is given by where is measured in minutes.

Find the velocity and the acceleration at . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ Calculates expressions for velocity and acceleration  ✓ Calculates velocity at t = 1  ✓ Calculates acceleration at t = 1 |

(b) A body moves according to the law .

Show the body is moving in SHM. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ Calculates expressions for velocity  ✓ Calculates expression for acceleration  ✓ Shows acceleration as -4x |

**Question 5 (7 marks)**

(a) A particle undergoing simple harmonic motion with a period of 5 seconds is observed to move in a straight line, oscillating 3.6 m either side of a central position. Determine the speed of the particle when it is 3 m from the central position. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ determines  ✓ substitutes into velocity equation  ✓ evaluates speed |

(b) Another particle moving in a straight line experiences an acceleration of ms-2, where is the position of the particle at time seconds.

Given that when , the particle had a velocity of 2 ms-1, determine the velocity of the particle when . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses appropriate form of acceleration  ✓ integrates  ✓ evaluates constant  ✓ states all possible values of v |

**Question 6 (8 marks)**

A particle moves along a straight line. Its displacement metres from a fixed point after seconds have elapsed is such that

If when , find

(a) as a function of (6 marks)

|  |
| --- |
| **Solution** |
| Since v > 0 when x > 2, and x starts at 3, x will always increase, hence |
| **Specific behaviours** |
| ✓ Separates variable  ✓ Integrates both sides  ✓ Determine value for c  ✓ Completes the square  ✓ Considers  ✓ Discounts –ve value and writes final solution |

(b) the distance travelled in the first 24 seconds. (2 marks)

|  |
| --- |
| **Solution** |
| Distance travelled is 4 metres |
| **Specific behaviours** |
| ✓ Calculates start and finish positions  ✓ Determines distance |

**Question 7 (13 marks)**

A large supermarket chain has decided to change its supplier for its home brand batteries. The batteries need to last for 9000 hours. Barry tests a random sample of 150 batteries from a supplier. The sample mean is found to be 9200 hours and the sample standard deviation 375 hours.

(a) Based on Barry’s sample, obtain a 95% confidence interval for , the population mean battery life. (4 marks)

|  |
| --- |
| **Solution** |
| Conf. Int: |
| **Specific behaviours** |
| ✓ Uses the correct mean  ✓ Calculates the correct standard deviation for the sample mean  ✓ Uses the correct z score  ✓ calculates the correct upper and lower limits |

(b) State whether each of the following statements is true or false. Provide reasons for your answer and state any assumptions.

(i) If another sample of 150 batteries is taken, then the sample mean will fall within the confidence interval found at part (a). (2 marks)

|  |
| --- |
| **Solution** |
| Statement is false.  The sample mean is based on another random sample. Only 95% of sample means will fall within the interval, so it is not certain. |
| **Specific behaviours** |
| ✓ states the statement is false  ✓ justifies the answer, ie not certain as it is random |

(ii) If a single battery is selected at random then there is a 95% chance that the battery will have a life that will fall within the confidence interval found at part (a). (2 marks)

|  |
| --- |
| **Solution** |
| Statement is false.  This is a single observation not a sample mean. The variation of single observations will be greater than the variation of the sample mean and hence more likely to fall outside the interval. |
| **Specific behaviours** |
| ✓ states the statement is false  ✓ justifies the answer, ie single observation has larger variation |

Barry’s supervisor, Steve, is a bit concerned. Steve isn’t certain that the life of the batteries are actually normally distributed, and hence the calculation of the confidence interval will not be correct.

(c) What should Barry say in response to Steve’s concerns? (2 marks)

|  |
| --- |
| **Solution** |
| Barry should tell Steve not to worry.  As the sample sizes are 150, it does not matter what distribution the parent population is as the distribution of the sample means will be approximately normal. |
| **Specific behaviours** |
| ✓ states that Steve’s worries are unfounded.  ✓ justifies the answer, ie large sample size will be normally distributed |

Before making a final decision on whether or not to go with the new supplier, Barry takes another sample, this time of 100 batteries. He finds that the standard deviation is 400 hours. Barry determines that the confidence interval for the population mean battery life is .

(d) Determine the confidence level, to the nearest 0.1%, used to calculate this interval.

(3 marks)

|  |
| --- |
| **Solution** |
| With  Hence a 92.0% confidence interval |
| **Specific behaviours** |
| ✓ Determines an error of 70  ✓ Solves for critical z score to yield this variation  ✓ Determines the correct confidence interval (to 0.1%) |